

Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.111 - Introductory Digital Systems Laboratory

Problem Set 1 Solutions

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Problem 1: Boolean Algebra Practice Problems (*Problem 1 was not graded.*)

- 1) $a + 0 = a$
- 2) $\bar{a} \cdot 0 = 0$
- 3) $a + \bar{a} = 1$
- 4) $a + a = a$
- 5) $a + ab = a(1 + b) = a$
- 6) $a + \bar{a}b = (a + \bar{a})(a + b) = a + b$
- 7) $a(\bar{a} + b) = a\bar{a} + ab = ab$
- 8) $ab + \bar{a}b = b(a + \bar{a}) = b$
- 9) $(\bar{a} + \bar{b})(\bar{a} + b) = \bar{a}\bar{a} + \bar{a}b + \bar{b}\bar{a} + \bar{b}b = \bar{a} + \bar{a}b + \bar{a}\bar{b} = \bar{a}(1 + b + \bar{b}) = \bar{a}$
- 10) $a(a + b + c + \dots) = aa + ab + ac + \dots = a + ab + ac + \dots = a$
- 11) $f(a, b, ab) = a + b + ab = a + b$
- 12) $f(a, b, \bar{a} \cdot \bar{b}) = a + b + \bar{a}\bar{b} = a + b + \bar{a} = 1$
- 13) $f[a, b, (\overline{ab})] = a + b + \overline{ab} = a + b + \bar{a} + \bar{b} = 1$
- 14) $y + y\bar{y} = y$
- 15) $xy + x\bar{y} = x(y + \bar{y}) = x$
- 16) $\bar{x} + y\bar{x} = \bar{x}(1 + y) = \bar{x}$
- 17) $(w + \bar{x} + y + \bar{z})y = y$
- 18) $(x + \bar{y})(x + y) = x$
- 19) $w + [w + (wx)] = w$
- 20) $x[x + (xy)] = x$
- 21) $\overline{(\bar{x} + x)} = x$
- 22) $\overline{(x + \bar{x})} = 0$
- 23) $w + (\overline{wxyz}) = w(1 + \overline{xyz}) = w$
- 24) $\bar{w} \cdot (\overline{wxyz}) = \bar{w}(\bar{w} + \bar{x} + \bar{y} + \bar{z}) = \bar{w}$
- 25) $xz + \bar{x}y + zy = xz + \bar{x}y$
- 26) $(x + z)(\bar{x} + y)(z + y) = (x + z)(\bar{x} + y)$
- 27) $\bar{x} + \bar{y} + xy\bar{z} = \bar{x} + \bar{y} + \bar{z}$

Problem 2: Karnaugh Maps and Minimal Expressions

It is best to approach each expression by first simplifying it using Boolean algebra learned Problem 1. If you can put the expression into a product-of-sums or sum-of-products form (not necessary minimal), then it is very easy to declare when the expression is a 0 or a 1, respectively.

$$\begin{aligned}
 1) & (a + b \cdot \bar{c}) + d \cdot (\bar{a} \cdot \bar{b} \cdot \bar{c} + a \cdot b) \\
 & = a + b \cdot \bar{c} + \bar{a} \cdot \bar{b} \cdot \bar{c} \cdot d + a \cdot b \cdot d \\
 & = a + b \cdot \bar{c} + \bar{a} \cdot \bar{b} \cdot \bar{c} \cdot d
 \end{aligned}$$

i)

| a | b | c | d | e x p r |
|---|---|---|---|------------------|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

ii)

ab ---MSP MPS

| cd \ ab | 00 | 01 | 11 | 10 |
|---------|----|----|----|----|
| 00 | 0 | 1 | 1 | 1 |
| 01 | 1 | 1 | 1 | 1 |
| 11 | 0 | 0 | 1 | 1 |
| 10 | 0 | 0 | 1 | 1 |

iii)

$$MSP = a + b\bar{c} + \bar{c}d$$

iv)

$$MPS = (a + \bar{c}) \cdot (a + b + d)$$

$$\begin{aligned}
2) & (\bar{d} + b \cdot \bar{c}) \cdot (c \cdot d + (\bar{a} + c) \cdot (\bar{c} + d)) \cdot (b + \bar{c}) \\
&= (\bar{d} + b \cdot \bar{c}) \cdot (c \cdot d + \bar{a} \cdot \bar{c} + \bar{a} \cdot d + \bar{c} \cdot c + c \cdot d) \cdot (b + \bar{c}) \\
&= (\bar{d} + b \cdot \bar{c}) \cdot (b + \bar{c}) \cdot (c \cdot d + \bar{a} \cdot \bar{c} + \bar{a} \cdot d) \\
&= (b \cdot \bar{d} + b \cdot \bar{c} + b \cdot \bar{c} + \bar{c} \cdot \bar{d}) \cdot (c \cdot d + \bar{a} \cdot \bar{c} + \bar{a} \cdot d) \\
&= b \cdot \bar{d} \cdot c \cdot d + \bar{a} \cdot b \cdot \bar{c} \cdot \bar{d} + \bar{a} \cdot b \cdot d \cdot \bar{d} + b \cdot \bar{c} \cdot c \cdot d + \bar{a} \cdot b \cdot \bar{c} + \bar{a} \cdot \bar{c} \cdot \bar{d} \\
&= \bar{a} \cdot b \cdot \bar{c} \cdot \bar{d} + \bar{a} \cdot b \cdot \bar{c} + \bar{a} \cdot \bar{c} \cdot \bar{d} \\
&= \bar{a} \cdot b \cdot \bar{c} + \bar{a} \cdot \bar{c} \cdot \bar{d}
\end{aligned}$$

i)

| a | b | c | d | e x p r |
|---|---|---|---|------------------|
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 |

ii)

| | | ab | | | |
|----|----|----|----|----|----|
| | | 00 | 01 | 11 | 10 |
| cd | 00 | 1 | 1 | 0 | 0 |
| | 01 | 0 | 1 | 0 | 0 |
| | 11 | 0 | 0 | 0 | 0 |
| | 10 | 0 | 0 | 0 | 0 |

--- MSP MPS

iii)

$$MSP = \bar{a} \cdot \bar{c} \cdot \bar{d} + \bar{a} \cdot b \cdot \bar{c}$$

iv)

$$MPS = \bar{a} \cdot \bar{c} \cdot (b + \bar{d})$$

$$\begin{aligned}
3) & \overline{(w \cdot y)} \cdot (\overline{w} + \overline{y} + z) \cdot (w + x + \overline{y}) \\
& = (\overline{w} + \overline{y}) \cdot (\overline{w} + \overline{y} + z) \cdot (w + x + \overline{y}) \\
& = (\overline{w} + \overline{y}) \cdot (w + x + \overline{y})
\end{aligned}$$

i)

| w | x | y | z | e x p r |
|---|---|---|---|------------------|
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 |

ii)

| | | wx | | | |
|----|--|--------|----|----------|----|
| | | ---MSP | |MPS | |
| yz | | 00 | 01 | 11 | 10 |
| 00 | | 1 | 1 | 1 | 1 |
| 01 | | 1 | 1 | 1 | 1 |
| 11 | | 0 | 1 | 0 | 0 |
| 10 | | 0 | 1 | 0 | 0 |

iii)

$$MSP = \overline{y} + \overline{w} \cdot x$$

iv)

$$MPS = (\overline{w} + \overline{y}) \cdot (x + \overline{y})$$

Problem 3: Karnaugh Maps with “Don’t Cares”

1)

| | | ab | | | |
|----|----|----|----|----|----|
| | | 00 | 01 | 11 | 10 |
| cd | 00 | X | 1 | 1 | X |
| | 01 | 1 | 1 | 0 | 0 |
| | 11 | 1 | 0 | 0 | 1 |
| | 10 | X | 0 | 0 | 1 |

Legend: ---MSP MPS

i) $MSP = \bar{c} \cdot \bar{d} + \bar{a} \cdot \bar{c} + \bar{b} \cdot c$

ii) $MPS = (\bar{b} + c) \cdot (\bar{a} + c + \bar{d})$

iii) Yes, the solutions are unique.

iv) Yes. We can tell that the $MSP=MPS$, without algebra, if their groupings on the Karnaugh map do not overlap and collectively cover the map.

2)

| | | ab | | | |
|----|----|----|----|----|----|
| | | 00 | 01 | 11 | 10 |
| cd | 00 | 1 | 1 | 1 | 0 |
| | 01 | 0 | X | 0 | 0 |
| | 11 | 1 | 0 | X | 1 |
| | 10 | 1 | 0 | 0 | 0 |

Legend: ---MSP MPS

i) $MSP = b \cdot \bar{c} \cdot \bar{d} + \bar{a} \cdot \bar{b} \cdot \bar{d} + \bar{b} \cdot c \cdot d$

ii) $MPS = (c + \bar{d}) \cdot (\bar{b} + \bar{c}) \cdot (\bar{a} + b + d)$

iii) Yes, the solutions are unique. But, consider a similar K-map with the following groupings:

| | | ab | | --- | MSP | ----- | MPS |
|----|----|----|----|-----|-----|-------|-----|
| | | 00 | 01 | 11 | 10 | | |
| cd | 00 | 1 | 1 | 1 | 0 | | |
| | 01 | 0 | X | 0 | 0 | | |
| | 11 | 1 | 0 | 1 | 1 | | |
| | 10 | 1 | 0 | 0 | 0 | | |

In this case, neither the MSP nor the MPS are unique since the following groupings are also valid. But in both groupings, the MSP=MPS.

| | | ab | | --- | MSP | ----- | MPS |
|----|----|----|----|-----|-----|-------|-----|
| | | 00 | 01 | 11 | 10 | | |
| cd | 00 | 1 | 1 | 1 | 0 | | |
| | 01 | 0 | X | 0 | 0 | | |
| | 11 | 1 | 0 | 1 | 1 | | |
| | 10 | 1 | 0 | 0 | 0 | | |

iv) Yes, the MSP equals the MPS.

Problem 4: DeMorgan's Theorem

$$\begin{aligned} 1) & \overline{\overline{(a+d)} \cdot \overline{(a+c)}} \\ & = \overline{\overline{(a+d)} + \overline{(a+c)}} \\ & = \overline{(a+d) + (a+c)} \\ & = \overline{a+c+d} \end{aligned}$$

$$2) \overline{a \cdot b \cdot c} \text{ (already simplified)}$$

$$\begin{aligned} 3) & \overline{\overline{a+d} \cdot \overline{a+c} \cdot \overline{c+d}} \\ & = \overline{(\overline{a \cdot d}) \cdot (\overline{a \cdot c}) \cdot (\overline{c \cdot d})} \\ & = \overline{(\overline{a \cdot d}) \cdot (a \cdot c) \cdot (\overline{c \cdot d})} \\ & = 0 \end{aligned}$$

Problem 5: Setup and Hold Times for D Flip-Flop

- 1) The setup time is twice the delay of the inverter. The hold time is zero.
- 2) The new memory element is a negative-edge triggered flip flop.
- 3) The setup time is $2t_{inv}$, the hold time is zero, and the clock to Q delay is $2t_{inv}$.